CSCI 3210:
Computational Game Theory

Computational Complexity of Games

Ref: AGT (Ch 2) and
Algorithm Design (Ch 8)
Gottlob et al. paper

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Decision problems

"Decision problems are too limited. Some computational problems are not easily expressed as decision problems . . . Yet the framework of decision problems turn out to be surprisingly expressive."

Arora & Barak, "Computational Complexity: A Modern Approach"

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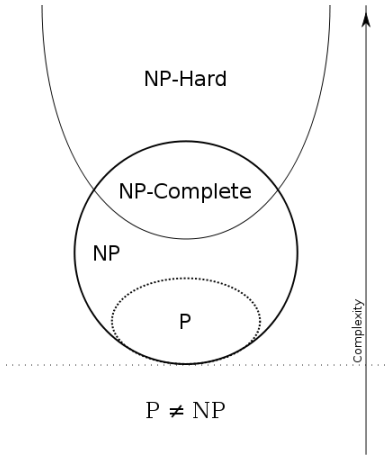
Complexity classes



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Complexity of computing NE

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Is there a pure-strategy NE (PSNE)?

- Games in "normal form" (with payoff matrices): polynomial-time
- But NP-complete for "graphical games"

Input size is important!

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PSNE in graphical games

Deciding whether a graphical game has a PSNE is NP-complete, even for max degree 3 and max # of actions 3.

(Gottlob et al., 2005)

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Complexity of MSNE

- Always exists (so not NP-complete!)
- Any twist is NP-complete

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Ref: AGT (Ch 2)

Theorem 2.3 (Gilboa and Zemel, 1989) *The following are NP-complete problems, even for symmetric games: Given a two-player game in strategic form, does it have*

- *at least two NASH equilibria?*
- *a NASH equilibrium in which player 1 has utility at least a given amount?*
- *a NASH equilibrium in which the two players have total utility at least a given amount?*
- *a NASH equilibrium with support of size greater than a given number?*
- *a NASH equilibrium whose support contains strategy s?*
- *a NASH equilibrium whose support does not contain strategy s?*
- *etc., etc.*

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MSNE computation

- OK, always exists, but how hard is computation?
- PPAD: Polynomial parity argument– directed case (Papadimitriou, 1994)
- MSNE computation in 2-player games is PPAD-complete

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PPAD-complete

- 4-player games (Daskalakis, Goldberg, and Papadimitriou, 2006)
- 3-player games (Daskalakis and Papadimitriou, 2005)
- 2-player games (Chen and Deng, 2005)

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